Recent Advances in Analysis of Laminated Beams and Plates, Part II: Vibrations and Wave Propagation

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A summary of the recent developments in the analysis of laminated beams and plates with an emphasis on vibrations and wave propagations in presented. First, a review of the recent studies on the free-vibration analysis of symmetrically laminated plates is given. These studies have been conducted for various geometric shapes and edge conditions. Both analytical (closed-form, Galerkin, Rayleigh-Ritz) and numerical methods have been used. Because of the importance of unsymmetrically laminated structural components in many applications, a detailed review of the various developments in the analysis of unsymmetrically laminated beams and plates also is given. A survey of the nonlinear vibrations of the perfect and geometrically laminated plates is presented next. It is seen that due to the bending-stretching coupling, the nonlinear behavior of the unsymmetrically laminated perfect and imperfect plates, depending upon the boundary conditions, may be hardening or softening type. Similar behavior also is observed for imperfect isotropic and laminated plates. Lastly, the developments in studying the wave propagation in laminated materials are reviewed. It is seen that a significant effort has been spent on developing appropriate continuum theories for modeling the composite materials. Some recent studies on the linear and nonlinear transient response of laminated materials also are described.

Introduction

THIS paper is a review of the recent advances in the analysis of laminated beams and plates. Laminated beams and plates are finding an increasing use in the mechanical, aerospace, marine, and other branches of engineering. Structural components made of laminated composite materials have a great potential for their utilization in a wide variety of applications such as in aircraft industry, automobile industry, sporting goods, offshore structures, and civil engineering-type applications.

A significant amount of research has been conducted on the vibration, buckling, and postbuckling analyses of laminated composite beams and plates. A review of some of these developments is given here. The review is restricted to the studies published in English. Thus, the review of the numerous studies in other languages is not being given here.

Specifically, the present paper, which is a sequel to the first paper,[1] will deal with the following developments: 1) linear vibrations of symmetrical plates, 2) analysis of unsymmetrically laminated plates, 3) linear and nonlinear vibrations of perfect and imperfect plates, and 4) wave propagation and transient response analysis.

It is felt that the present survey paper will be of interest to the technical community engaged in the analysis and design of composite structures.

Linear Vibration Analysis

A significant number of investigations have been conducted on the linear vibrations of laminated beams and plates. Both analytical (closed-form, Galerkin, Rayleigh-Ritz method) and numerical methods have been used. An extensive review of the vibrations of thick (isotropic and laminated) beams was given by Kapania and Raciti.[2] A review of the technical literature on the vibration response of anisotropic plates was given by Leissa[3] and on dynamic behavior of laminated composites by Bert.[4] A review of the application of the finite-element method to study the vibration response of plates was given by Reddy.[5,6] A review of some other recent studies on the vibration of symmetrically laminated plates is being given here. A review of the studies on the vibrations of the unsymmetrically laminated plates is given in the next section where a detailed review of studies on the analysis of the unsymmetrically laminated plates will be given.

Reddy and Kuppusamy[7] performed a vibration analysis of laminated plates using a three-dimensional solution. Bhimaraddi and Stephens[8] presented results of vibrations of thick laminated plates using a higher-order theory. The effect of transverse shear on the cylindrical bending, vibration, and buckling of laminated plates was presented by Stein and Jegley.[9] Transverse vibrations of nonuniform rectangular orthotropic plates was given by Tomar et al.[10]

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Sadasiva Rao and Singh[11] studied the vibrations of corner-supported thick composite plates. Dawe and Craig in a series of papers[12–14] considered the effect of shear deformation and in-plane stresses on thick symmetrically laminated plates. The plates had nonvanishing bending-twisting shear coupling. The results were presented for homogeneous, unidirectional composites with the fibers off-axis, relative to the sides of the plate.

To overcome the aforementioned limitations, a free-vibration analysis using the Galerkin method and incorporating three different boundary conditions was conducted by Bowlus et al.[15] The authors considered the effects of transverse shear deformation and rotatory inertia. Bending-twisting shear coupling terms also were included in the study.

The governing equations based on Mindlin-type laminated plate equations were used.[15–17] These equations were given to be[15]

$$M_{x,x} + M_{xy,y} - Q_x = I\ddot{\psi}_x \tag{1a}$$

$$M_{xy,x} + M_{y,y} - Q_y = I\ddot{\psi}_y \tag{1b}$$

$$Q_{x,x} + Q_{y,y} + q = \rho\ddot{w} \tag{1c}$$

where ψ_x and ψ_y are, respectively, the rotation of a line perpendicular to the midplane due to bending with respect to the y- and x-axes; w is the transverse deflection; the comma denotes the derivative with respect to x or y; and the dot denotes the derivative with respect to time. The bending moments M and shear forces Q are related to ψ and $w,_x$ and $w,_y$ and are given by

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \psi_{x,x} \\ \psi_{y,y} \\ \psi_{x,y} \end{Bmatrix} \tag{2a}$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = K \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} \begin{Bmatrix} w,_y + \psi_y \\ w,_x + \psi_x \end{Bmatrix} \tag{2b}$$

where K is a shear correction factor. The moments and shear forces were given in Part I.[1] Relations for D_{ij} and A_{ij} were also given in Part I.[1]

Equations (1) were solved by Bowlus et al.[15] using the Galerkin method. Two different graphite-epoxy symmetric plates with $[0/90]_s$ and $[\pm 45]_s$ orientations were used in the analysis. The three different boundary conditions considered for each plate were: simply supported, clamped, and two opposite sides clamped/two opposite sides simply supported. Significant reduction in natural frequencies were observed due to the shear deformation. The effect of shear deformation was found to depend upon boundary conditions: it was most significant for plates with clamped conditions as compared to the plates with simply supported conditions. The influence of rotatory inertia was found to be negligible.

The vibrations of thick laminated rectangular plates with free edges were studied by Sivakumaran.[18] Vibrations of clamped trapezoidal plates having rectangular orthotropy were studied by Narita et al.[19] and those of clamped orthotropic skew plates were studied by Skata and Hyashi.[20] Efforts also have been made to study the vibration of circular and elliptic plates. Narita[21] studied the free vibrations of completely free continuous polar orthotropic annular and circular plates having polar orthotropy. The same author, in a later study[22] also studied the vibrations of continuous annular and circular plates. Vibrations analysis of composite annular plates by using an integral equation formulation was performed by Srinivasan et al.[23] Vibrations of annular plates with rectangular orthotropy (as opposed to polar orthotropy) were studied by Bianchi et al.[24]

Tomar and Gupta[25] studied the vibrations of an orthotropic elliptic plate of nonuniform thickness and temperature. Lee et al.[26] studied the free vibrations of composite rectangular plates with rectangular cutouts.

It should be noted that free-vibration analysis of rectangular plates using other higher-order shear-deformation theories also have been performed. This includes the works of Reddy[27] using a higher-order shear-deformation theory given by Reddy,[28] Di Sciuva,[29] Bhimaraddi and Stephens,[8] and Doong and Chen.[30] Doong and Chen[30] also considered the effect of initial stresses on the natural frequencies.

Sayir[31] presented theoretical and experimental results on the dynamic behavior of composite beams, plates, and shells. An asymptotic expansion of the basic three-dimensional equations of linear elasticity with respect to two small geometric parameters was performed to derive consistent sets of equations with well-defined ranges of validity. The results were obtained for flexural waves in axially reinforced composite beams, dynamic flexural behavior of sandwich plates, flexural waves in unidirectionally reinforced thin plate, and axisymmetric waves in a fiber-reinforced cylindrical shell. The theoretical results were compared with experimental results.

Schmidt[32] studied the transverse vibrations of an infinite Timoshenko beam by solving the governing partial differential equations using the Riemann theory. In this theory, solution of arbitrary initial-value hyperbolic problems is obtained by using a simple quadrature involving the initial data and a set of functions called adjoint functions that do not depend upon initial data. Schmidt obtained the adjoint functions for the Timoshenko beams to study the wave propagation in these beams.

Unsymmetric Laminates

Most of the research done on laminated composite plates is restricted to the special case of symmetric laminates. Symmetric laminates are simpler to analyze since they do not exhibit bending-stretching coupling that is characteristic of unsymmetric laminates. There are some important reasons for designing and analyzing structural elements with such coupling responses:

1) As suggested by Minich and Chamis,[33] it would be possible to select coupling effects that will provide built-in self-damping mechanisms when subjected to dynamic responses.

2) Warpage of structural elements composed of composite materials is largely due to coupling responses that result from fabrication errors in orienting the different plies.

3) The bending-stretching coupling will cause unsymmetric laminates to warp out-of-plane during the curing process as discussed in following lines.

4) Cross-coupling parameters that describe the degree of coupling between bending curvature and twist rate for a laminated composite are used in aeroelastic tailoring to control mode shapes and frequencies of high-aspect-ratio lifting surfaces.[34,35]

For symmetric laminates, the classical lamination theory (CLT) has been used successfully in most cases. Classical lamination theory assumes that the Kirchhoff's hypothesis is valid for the laminate and that the through-the-thickness effects and geometric nonlinearities are negligible. The CLT assumptions and the integration of the laminae constitutive equations lead to the well-known A, B, and D matrices. For symmetric laminates, the B matrix is identically zero, and therefore the inplane and out-of-plane effects are uncoupled. With the increasing application of unsymmetrically laminated plates for certain applications such as jet turbines fan blades, helicopter blades, heat shields, and the skin of aircraft structures, more and more research effort has been devoted to their analysis.

Because of the bending-stretching coupling, it would be impossible to pull on unsymmetric laminates without at the same time bend and/or twist the laminate. Although the bending-stretching coupling is present only in unsymmetric laminates, the stretching-shearing and the bending-twisting coupling may characterize both symmetric and unsymmetric laminates. The effect of the stretching-shearing coupling is to induce shear when a stretching force is applied to the laminate and vice versa. The effect of bending-twisting coupling is to induce twisting when a bending moment is applied to the laminate and vice versa.

For unsymmetric laminates, the coupling effects will introduce odd-power derivatives in the governing differential equations, and the boundary conditions become more complicated since the in-plane and transverse effects must be considered simultaneously. Therefore, different approaches than those used for symmetric laminates must be considered.

In his work on unsymmetric laminates, Hyer[36] noted that CLT predicts the room temperature shape of such laminates to be a saddle shape; however, in many instances, experimental observations show that the actual room-temperature shape is cylindrical. It is the bending-stretching coupling that induces originally flat laminates to warp out-of-plane during the curing process. A knowledge of this warping behavior could be used to obtain shapes with desired curvatures such as the skin of a wing. The coupling effects introduce geometric nonlinearities that must be included in the analysis of unsymmetric laminates.

Earlier work on the nonlinear coupling effects in unsymmetrically laminated plates was given by Reissner and Stavsky.[37] A conclusion common to all of the literature reviewed here on this topic is that the bending-stretching coupling effects increase transverse deflections, decrease fundamental frequencies of vibration, and decrease buckling loads relative to the analogous anisotropic homogeneous plate.

Whitney and Leissa,[38] using the classical Kirchhoff plate theory in conjunction with the nonlinear terms in von Kármán's theory, developed the governing equations for laminated anisotropic plates with coupling effects. Closed-form solutions to the linearized equations were obtained for transverse deflection, flexural vibration, and buckling loads. An analytical solution for a simply supported anisotropic plate was carried out by the same authors in Ref. 39.

One of the first linear analyses for the free vibration of unsymmetrically laminated plates with clamped edges was presented by Bert and Mayberry.[40] Using Kirchhoff's hypothesis, and minimizing the Lagrangian, in Hamilton's principle, by the Raleigh-Ritz method, the analysis was reduced to the solution of a standard eigenvalue problem.

As previously mentioned, the coupling effects lead to complications in the boundary conditions. This problem was analyzed and discussed by Whitney.[41] Whitney investigated the effect of boundary conditions on the bending, vibrations, and buckling of unsymmetrically laminated plates. By analyzing five sets of boundary conditions, he noted that the coupling effect is essentially independent of boundary conditions, but it is mainly a function of the longitudinal and transverse stiffnesses ratio of each ply and of the number of layers. He also noted that for certain orientations of antisymmetric, angle-ply laminates the membrane effect can greatly influence the response of the plate. These results also were obtained in an earlier study by the same author.[42] Some important conclusions that can be derived from Whitney's papers are as follows:

1) Coupling effects can be reduced by increasing the number of layers for the particular cases of angle-ply and cross-ply laminates.

2) For uniformly loaded laminates with aspect ratios greater than 2, the maximum deflection of the plate does not occur at the center but along a circle of small radius about the center.

3) Coupling can increase maximum deflection by as much as 300% compared to analogous uncoupled plates.

Because of the complexities in deriving closed-form solutions for anisotropic plates with coupling effects, Ashton[43] used an approximate method as a simpler approach to the problem. In his formulation, Ashton used a "reduced stiffness matrix" to uncouple the terms in the total potential energy, thus the coupling present in the general differential equation is due to compatibility equations only. This type of solution leads to very good results for simply supported plates. The applicability of this method for the general case of unsymmetrically laminated plates is not demonstrated, however.

The analysis of unsymmetric laminates is extended to the special case of cross-ply simply supported plates in the work done by Jones.[44] His results led to the same conclusions obtained by earlier investigators, but no previous work considered unsymmetric cross-ply laminates. However, an additional consideration obtained from his work is that even though the coupling effects rapidly die out in antisymmetric laminates as the number of layers is increased, they decrease very slowly for unsymmetric laminates. Therefore, coupling effects need to be considered also for unsymmetric laminates with a large number of layers.

A common aspect of all the papers discussed so far on the analysis of unsymmetrically laminated plates is that they are all restricted to specified boundary conditions, thus do not allow for the analysis of general boundary conditions unless proper changes are made. Lin and King[45] used a different approach so that general boundary conditions can be analyzed. Their analysis gave exact values for the vibration frequencies when a pair of opposite edges were simply supported, and for other boundary conditions the eigenvalues were estimated using Bolotin's method.

The analysis of laminated plates having arbitrary boundary conditions is getting increasing attention in recent days. Baharlou and Leissa[46] presented a general method capable of analyzing generally laminated composite thin plates having arbitrary edge conditions such as simply supported, clamped, or free. The procedure was an extension of the Ritz method. The method was described in somewhat greater detail in Part I. Khdeir[47] studied the free vibrations of antisymmetrically angle-ply laminated plates with various boundary conditions. The method was based on a generalized Levy-type solution[48] considered in conjunction with the space-state concept.[49] The author considered the shear-deformation effect using Mindlin-Reissner type plate equations. The solution was applicable to only those plates for which the two opposite edges are simply-supported. The results were compared with both analytical and finite element results[50-51] available in the literature for all edges simply supported.

Finally, the vibrations of arbitrarily laminated plates with arbitrary boundary conditions can be studied using the finite-element method. This approach was taken by Kapania and Yang[52] for thin plates, by Putcha and Reddy[50-51] for thick plates, and by Kapania and Raciti[2] for thick beams.

The research conducted by Sharma et al.[53] was motivated by the fact that no previous attempt has been made to find how the degrading coupling effects could be minimized. Their results showed that by choosing proper lamina thicknesses the coupling effects can be completely eliminated for alternating laminates and partially reduced for other types.

The recent work by Kamal and Durvasula[54] differs from the studies so far discussed in the fact that a shear-deformation plate theory, incorporating all possible laminate boundary conditions, is used for studying the free vibrations of unsymmetric laminated plates. The effect of thickness ratios of the layers for antisymmetric and unsymmetric plates also are discussed as a way to reduce coupling effects.

Experimental and analytical data for the vibration characteristics of cantilevered, unsymmetrically laminated, boron-epoxy panels were presented by Thornton.[55] The effect of the panel asymmetry was to decrease the frequency of vibration. Experimental studies also were conducted by Rasskazov and Sokolovskaya[56] to study the stressed-strained state and natural oscillations of symmetric and unsymmetric plates.

Vibrations and Aeroelastic Behaviors of Laminated Wings

In recent years, considerable effort is being spent on accurately and efficiently analyzing the wings that have skin panels made of composite materials. The emphasis is on representing a wing by an equivalent plate or beam. Giles[57] presented an equivalent plate model for analyzing aircraft wing structures with general planform such as cranked wing boxes. Multiple trapezoidal segments were used to represent such planforms. A Ritz-solution technique that employs global displacement functions that span all the segments was used. Highly accurate results were obtained for natural frequencies of a wing box. The formulation was restricted to symmetrically laminated wings. This restriction was later removed by the same author in Ref. 58. This formulation has been extended by Bergen and Kapania[59] to study the dynamic aeroelastic behavior and its sensitivity to various geometric shape parameters. Castel and Kapania[60] have extended the previous work of Kapania and Raciti[2] to study the vibrations and aeroelastic behavior of arbitrarily swept, and laminated box wings with damaged skins. Finally, it is noted that vibrations and aeroelastic behavior of laminated structures also have been studied by Chen and Dugundji[61] and Weisshaar and Bohlman.[62]

Nonlinear Vibrations

The geometrically nonlinear analysis of laminated anisotropic plates recently has received considerable attention. As compared to isotropic plates, the analysis of laminated plates is complicated by the fact that even for infinitesimal amplitudes the bending-stretching coupling will affect the response of the plate. A detailed study on the nonlinear vibrations of plates, both isotropic and anisotropic, can be found in the text by Chia.[63] Some early studies were presented by Bennett,[64] Bert,[65] Chandra and Basava Raju,[66,67] and Wu and Vinson.[68] Reddy[69] studied the large-amplitude vibrations of rectangular composite plates with rectangular cutouts using the finite-element method.

A limited amount of material is available on the nonlinear vibration of laminated composite beams. A recent survey on the geometrically nonlinear analysis of isotropic beams using analytic and finite-element methods was presented by Sathyamoorthy.[70,71] A simple one-dimensional beam element for the nonlinear vibration analysis of unsymmetrically laminated beams was given by Kapania and Raciti.[2]

Chen and Lin[72] studied the large-amplitude vibrations of initially stressed bimodulus thick plates. The nonlinear governing differential equations were solved using the Galerkin method in space for a simply supported rectangular bimodulus plate subjected to a combination of a pure bending stress and extensional stress in the plane of the plate. The Runge-Kutta method was used to solve the nonlinear equations in time. Effects of various parameters on the large-amplitude vibrations also was studied.

Singh and Sadasiva Rao[73] studied the nonlinear vibrations of thick composite plates. The full nonlinear terms in addition to the usual von Kármán nonlinearities were included. These nonlinearities were found to affect the nonlinear frequencies of moderate thick plates by as much as 30–63%. The frequency ratio was found to generally increase with amplitude ratio for angle-ply and cross-ply laminates, while a decreasing trend was observed in the case of thick plates. For some geometric parameters, the nonlinear effects were greater for cross-ply laminates than those for angle-ply laminates at higher amplitude ratios.

Rajagopal et al.[74] studied the large deflection and nonlinear vibrations of multilayered sandwich plates using a quadratic rectangular finite element that included all the nonlinear terms in the strain-displacement relations. Results were presented for simply supported and clamped boundary conditions and a combination of those boundary conditions. In general, it was observed that a plate with clamped boundary yields least nonlinearity. It is noted that such a behavior for the case of laminated beams also was observed by Kapania and Raciti.[2] The nonlinearity was found to be predominant in the case of plates with a greater number of layers.

Hill and Majumdar[75] presented results for thermally induced large-amplitude vibrations of viscoelastic plates and shallow shells. The concept of isoamplitude contour lines in conjunction with Galerkin's method was used to discretize the governing nonlinear equations in space and the resulting equations in time were solved for appropriate time functions.

Effect of Geometric Imperfections

During fabrication of both isotropic and laminated composite plate structures, it is possible that certain deviations between the actual and intended (desired) shapes may occur. These deviations or the so-called geometric imperfections are generally quite localized and may significantly alter the structural behavior of these plates. The works of Kapania and Yang[52] and Saigal et al.[76] allow for the analysis of laminated plates and shells with arbitrary imperfections. Yamaki and co-workers studied the effect of geometric imperfections and initial edge in-plane displacements on the postbuckling, small-amplitude free vibrations, and large-amplitude vibrations of rectangular plates.[77,78] In addition to theoretically predicted responses, Yamaki and co-workers also observed experimentally a variety of nonsymmetric and nonperiodic responses in connection with the internal resonance, combination resonance, and dynamic-snap-through phenomena.

Studies on the effect of geometric imperfections on vibrations of simply supported flat plates under in-plane uniaxial or biaxial compression were conducted by Hui and Leissa[79] using analytical methods and by Ilanko and Dickinson[80–81] using both theoretical and experimental methods. It was determined that significant increase in the natural frequencies may occur in the presence of imperfections. Effect of geometric imperfections on the large-amplitude vibrations of the rectangular plates with hysteresis damping was studied by Hui.[82] Research on the nonlinear vibrations of simply supported and clamped circular plates in the presence of imperfections was conducted by Hui.[83]

Recently, Chia studied the large-amplitude vibration of unsymmetric angle-ply rectangular plates on elastic foundation having the varying rotational edge constraints.[84] Celep[85] studied the effect of shear and rotatory inertia on the nonlinear vibrations of the imperfect plate. Effect of geometric imperfections on the postbuckling behavior of imperfect laminated plates was studied by Hui.[82] Hui[86] also studied the effect of these imperfections on the nonlinear vibrations of the biaxially compressed antisymmetric angle-ply rectangular plates. Additional work on the effect of geometric imperfections on the nonlinear vibrations of antisymmetrically laminated angle- and cross-ply simply supported and clamped rectangular thin plates was presented by Hui.[87] It was observed that the presence of imperfection amplitudes of the order of only half the total laminated plate thickness may significantly alter the vibration frequencies and change the large-amplitude vibration behavior from the hard-spring to soft-spring behavior. Similar behavior also was observed by Kapania and Yang.[52]

This effect also was observed by Kapania and Raciti[2] in the nonlinear vibrations of unsymmetrically laminated beams. Because of the bending-stretching coupling present in these beams, the nonlinearity may change from a hardening type to a softening type for certain boundary conditions. The bending-stretching coupling induces a quadratic nonlinearity in addition to the cubic nonlinearity present in the system. It is the presence of the quadratic nonlinearity that is responsible for changing the response behavior from hardening type to softening type. For a detailed review, the reader is referred to a recent paper by Pandalai.[88]

Some recent studies on the nonlinear oscillations of laminated plates were conducted by Sivakumaran and Chia,[89] who studied the large-amplitude oscillations of unsymmetrically laminated plates including shear, rotatory inertia, and transverse normal stress; by Reddy,[90] who studied the effect of coupling on the transient response of laminated plates; and by Bhimaraddi,[91] who studied the influence of in-plane forces on the nonlinear vibrations of rectangular plates.

Finite-Element Analysis

The finite-element method has extensively been used for studying nonlinear vibrations. A review of various finite-element developments on geometrically nonlinear analysis of plates and shells was given by Kapania and Yang,[92] Saigal et al.,[76] Bushnell,[93] Reddy and Chao,[94] and Yang and Han.[95] The finite-element method has been employed by Mei,[96] and Rao et al.[97,98] Lau et al.[99,100] studied the nonlinear vibrations of thin elastic plates using a triangular plate element with 15 stretching and bending nodal displacements based on an incremental modified discrete Kirchhoff theory. Finally, Liu and Reddy[101] studied the nonlinear vibration of laminated rectangular plates using a refined shear deformable finite element based on a higher-order theory.[102]

The effect of rotatory inertia and shear deformation on the geometrically nonlinear behavior of composite plates have been studied by Yu and Lai,[103] Wu and Vinson,[68,104] Sathyamoorthy,[105] and Sivakumaran and Chia.[106] Results from those studies[68,104,105] indicate that the effect of transverse shear deformation decreases with increasing amplitude and that the rotatory inertia effect is small compared to the shear effect.

Wave Propagation

Investigations in the area of wave propagation in laminated medium had been conducted by geologists and physicists interested in the study of the propagation of seismic waves. The increasing use of laminated composites in the industry has led to a more elaborate area of research and new theories. Experimental and numerical techniques have been developed for studying wave propagation in laminates. These studies are also important for nondestructive evaluation of the composite materials.[107-114]

The composite materials, due to their microstructural heterogeneity, may exhibit response phenomena that are not observed for homogeneous materials. One such phenomenon is wave dispersion. This phenomenon, which is important both from the standpoint of direct response analysis and nondestructive (ultrasonic) testing, was demonstrated experimentally by Tauchert and Guzelsu,[115] and by Sutherland and Lingle.[116] The study of wave dispersion requires a higher-order continuum description, and several models are available in the literature. Only a brief description of some of the higher-order continuum theories is given below. For a detailed review, the reader is referred to a 1980 paper by Ting,[117] and for the review of some of the Russian literature on buckling and wave propagation, the reader is referred to a paper by Babich et al.[118]

Approximate Continuum Theories

Several different approaches of constructing mathematical models for representing the composite materials to study the dynamic response of composites have been proposed.

Sun et al.[119-120] and Achenbach et al.[121] studied dispersion characteristics for a certain class of free time-harmonic waves propagating in the direction of layering. In this work, they also studied the dispersion characteristics to check the validity of the "effective modulus theory." They showed that in the case of high ratio of layer stiffnesses, the lowest two modes show substantial dispersion even for very long wavelengths. Furthermore, they proposed a continuum theory that allows the introduction of the effective stiffness of both the reinforcing layers and the matrix layers. Using this theory, dispersion curves for harmonic waves propagating parallel to and normal to the layering were obtained. This theory called the "effective stiffness theory" has been studied and applied by various other authors for linear[122-124] and nonlinear materials.[125,126] Based on the continuum theory of mixtures, Bedford and Stern[127] proposed a theory for composite materials. When their solutions were compared with the classical solutions for longitudinal wave propagation in laminated composites, some aspects of the dynamic behavior were predicted extremely well.

Peck and Gurtman[128] presented a theoretical analysis of the geometric dispersion of transient stress waves in a linearly elastic laminated composite using the technique based on sinusoidal modes. The propagation of an initially sharp plane compressive stress pulse in a medium of linearly elastic composite was analyzed by Ting and Lee.[129] The damage caused by such a pulse depends on the attenuation due to scattering and absorption and also depends on the effect of tensile stresses associated with the reflection at the back free surface of the slab. Their approach was based on ray tracing and associated wave-front analysis of geometrical optics, as even in the case of elastic waves with two basic speeds in each material, the stress amplitude is given exactly by the laws of geometric optics associated with the propagation of irrotational waves.

Sun[130] presented a two-dimensional theory for laminated plates deduced from the three-dimensional continuum theory for a laminated medium. Besides the usual gross displacements, new kinematic variables termed as local deformations were included. Sve[131] studied the effect of propagation angle on the dispersion of time-harmonic waves traveling in a laminated media composed of alternating layers of hard and soft material. It was shown that this investigation provided an exact solution. Hegemier and Nayfeh[132] provided a continuum theory for propagation normal to the layers of a laminated composite with an elastic, periodic microstructure. Their continuum construction procedure was based on an asymptotic scheme that assumes that dominant signal wavelengths are large compared to the composite microdimensions. Nayfeh and Gurtman[133] developed a continuum mixture theory to study harmonic and transient shear wave motions in laminated wave guides. Waves polarized both transversely (SV) and horizontally (SH) were considered. Hegemier and Bache[134] extended the continuum theory with microstructure for wave propagation in laminated composites (proposed in the earlier works[132] concerning the wave propagation, normal and parallel to the laminates) to the general two-dimensional case. Nayfeh[135] derived the dispersion relation for time-harmonic waves propagating normal to the layers of a multilayered periodic composite.

Murakami,[136] and Murakami and Akiyama[137] presented a mixture theory for wave propagation in angle-ply laminates. In an effort to develop the continuum model for elastic angle-ply laminates, an asymptotic mixture theory with multiple scales was presented.

Murakami and Hegemier[138] presented a mixture model for unidirectionally reinforced composites using an asymptotic scheme with multiple scales and application of Reissner's new mixed variational principle.[139] A unique feature of the mixture model was that it was capable of simulating harmonic wave dispersion in laminated composites more accurately than effective stiffness theories. Comparison of the mixture model predictions with available experimental data on dispersion of harmonic waves was presented for boron/epoxy and tungsten/aluminum composites. Murakami and Hegemier[140] presented a nonlinear constitutive model with a considerably small number of displacement variables. The model was based on an asymptotic mixture theory using a displacement-based variational method. They made comparison of the constitutive predictions with finite-element results for a graphite-aluminum composite and also with other available experimental data of axial thermal expansion of a tungsten-copper composite under thermal cycles.

Duan[141] proposed a new method for the analysis of plane wave propagation in a periodically layered, elastic, nonhomogeneous composite body. Within each layer, the variation of material properties depended on the spatial coordinates.

In sum, for the case of most realistic composites, it is true that exact description of the static and dynamic cases generally cannot be obtained. As such, many approximate models have been developed, the complexity of which depends on the need for detailed information. In particular, there has been effective modulus theories, the effective stiffness theory, variational methods theories, continuum mixture theories, and continuum theories with microstructure based upon asymptotic expansions.

Studies in Wave Propagation

Viano and Miklowitz[142] determined the response of a symmetrically layered elastic plate subjected to a symmetric step normal line face load. The principal objective of this work was to understand transient response. The solution was based on the equations of motion from linear elasticity. Drumheller and Bedford[143] studied wave propagation in laminates using a second-order microstructure theory. A generalization of the simpler microstructure theory, developed earlier for laminates by Sun et al., was used to analyze steady-state plane wave propagation. This new version incorporates higher-order thickness variations in the displacement functions and includes restrictions on both displacement and stress at the laminate interfaces. Longscope and Steel[144] developed an approximate solution for one-dimensional pulse propagation in a medium with an analytic wave speed when the ratio of the pulse length to a characteristic length of the wave speed variation was small.

It is noted that the dispersion characteristics for harmonic wave propagation have been analyzed using Floquet's theory in conjunction with the elasticity solution by Delph et al.[145-147]

Shah and Datta[148] presented a stiffness method using the continuity of displacement and traction at the interfaces of a periodically laminated composite medium in conjunction with the Floquet's theory to study the harmonic wave propagation in a layered composite. Each lamina was allowed to have anisotropic properties as opposed to the previous studies that treated each lamina as an isotropic material. Also, the antiplane and plane cases were considered separately. Results were compared with the previous work of Delph et al.[145,147] for the isotropic case. Results were presented for a boron fiber-reinforced aluminum composite.

Podlipenets and Shul'ga[149] studied the love waves in regularly laminated isotropic composites, at a microstructural level in accordance with the three-dimensional theory of elasticity. Karim-Panahi[150] employed the Floquet's theorem to study the dispersion relation of propagation of harmonic shear waves (SH waves) in an infinite elastic, periodically triple-layered media.

Ghosh et al.[151] pointed out the need to take into account the finite extent of the media, multiple reflections, and physically realizable boundary conditions. They presented a method based on the transfer matrix method that does not depend upon the Floquet theory. Results were presented for attenuation and maximum tensile stresses developed in periodically layered elastic composites, subjected to rectangular compressive pulses.

Podlipenets[152] studied the propagation of harmonic waves in orthotropic materials with a periodic structure. General dispersion equations for an unbounded orthotropic-layered periodic media were derived in a three-dimensional formulation using the theory of differential equation with periodic coefficient. Podlipenets and Shul'ga[153] studied the propagation of harmonic shear waves (SH waves) in an infinite plate consisting of an arbitrary but finite number of layers. That is, the effect of the outer boundary of the medium was considered. Green and Milosavljevic[154] examined the extensional waves propagating in an infinite plate of transversely isotropic material with the axis of transverse isotropy lying in the plane of the plate. Attention was restricted to the waves propagating in the direction of transverse isotropy. Baylis and Green[155] considered the flexural waves propagating parallel to the axial direction in the core and for waves propagating parallel to the axial direction in the outer layer. In a sequel, Baylis and Green[156] also studied the flexural waves in a fiber-reinforced transversely isotropic continuum, for which the extensional modulus along the axis of transverse isotropy is very much greater than the shear modulus. Three plates were bonded together to form a symmetric laminate with the fiber direction in the outer layers at right angles to the fiber direction in the core. Dispersion curves were obtained for the fundamental mode of flexural waves propagating in the plane of the plate at various angles to the core fiber direction.

Experimental Studies

Clark et al.[157] conducted a photoelastic study of the propagation of high-frequency stress waves in bars and plates. They used a piezoelectric transducer for the application of a decaying sinusoidal wave. It was found that good correlation existed with theoretical computations for phase and group velocities. Whittier and Peck[158] made experimental study on the propagation of dispersive pulse as transient stress wave in laminated composites and compared their results with the theoretical predictions. Robinson and Leppelmeier[159] conducted experiments to verify the dispersion relations for layered composites. The propagation of ultrasonic shear waves normal to the lamination in the case of a steel/copper composite was studied, and the existence of allowed propagation frequencies (bands) and forbidden frequencies (gaps) had been observed for three bands and two gaps. Sve and Okubo[160] conducted experiments on the propagation of a stress wave in three laminated samples each having lamination angles of 0, 43, and 90 deg, respectively.

Kinra and Anand[161] determined experimentally the longitudinal and the shear-wave velocities in a random composite. An extensive investigation of the problem was carried out by using thirty-five specimens. They presented some evidence that indicated that the wave propagation may be occurring along two separate branches 1) the lower or the acoustical branch and 2) the upper or the optical branch.

Acoustic–Elastic Effect

Bonilla and Keller[162] studied small-amplitude elastic wave propagation in an aggregate of randomly oriented elastic grains, each with cubic symmetry. They computed the velocity and the attenuation coefficient for both coherent longitudinal waves and coherent shear waves by using a smoothing method. From the results, they calculated the acousto-elastic coefficients that relate the change in velocity to the applied stress and applied strain. The equations of acousto-elasticity were formulated by Pao and Gamer[163] in both natural and initial frames of reference and were applied to investigate the propagation of ultrasonic waves in orthotropic elastic solids with initial stresses. Vezzeti[164] developed a method of computing the elastic displacement in an anisotropic elastic medium (crystal) when the medium is excited at its surface by a given displacement or stress. In particular, it was shown that arbitrary boundary conditions generally induce both diagonal and off-diagonal stresses and longitudinal, as well as transverse, displacements. The author also developed a perturbation theory for narrow-angle beams.

Delsanto and Clark[165] described a perturbation method for the investigation of propagation of Raleigh waves on the surface of a homogeneous anisotropic initially deformed material plate. The formalism is quite general and also can be applied when other small effects, like slight temperature changes or external magnetic fields, affect the Raleigh wave propagation velocity.

Green and Milosavljevic[154] studied the propagation of extensional waves in an infinite plate of transversely isotropic material with the axis of transverse isotropy lying in the plane of the plate. They obtained expressions for the phase velocity as a function of wavelength and also for the variation of stress through the thickness of the plate. Corresponding results also were obtained for the idealized material that is inextensible in the direction of transverse isotropy. They came to the conclusion that the inextensible material serves to model longitudinal wave propagation in the fiber-reinforced material only for a restricted range of wavelengths and, in general, it is necessary to employ a transversely isotropic model that includes extensibility in the fiber direction. Ramsakaya and Shul'ga[166,167] studied the propagation of elastic waves in an orthotropic cylinder. The solution proposed by these authors makes it possible to fully study the exact spectrum of the phase and group velocities of elastic waves for a hollow cylinder made of a cylindrically orthotropic material.

Linear and Nonlinear Transient Analysis

A number of analytical studies have been performed on the linear transient analysis of plates. Sun and Whitney[168,169] studied the linear forced response of composite plates using the method of orthogonality of modes—the so-called modal superposition scheme. This method also has been used by Khdeir and Reddy[170] to study the dynamic response of composite laminates according to a third-order shear-deformation theory. These authors also employed a state-variable approach (often used in control theory) to obtain some exact solutions for the transient response of symmetrical cross-ply laminates using a higher-order theory advanced previously by Reddy.[171] Other investigations on the linear transient response using higher-order theories have been conducted by Bhimaraddi.[172,173] The effect of initial stresses on the transient response has been studied by Sun and Chatopadhyay[174] and by Sun and Whitney[175] using the method of modal superposition.

A review of some of the developments on linear transient response has been given by Khedir and Reddy[170] and also by Bhimaraddi.[172]

A majority of the analytical studies have been conducted without considering the nonlinear effects. The nonlinear transient elastic response of plates subjected to arbitrary transient loads is an important design consideration for components of missiles, spacecraft, pressure vessels, nuclear reactors, etc. These plates may undergo moderately large deflections of the order of the plate thickness under extreme dynamic loadings. So it is important to include nonlinear effects in the analysis.

An analytical method for determining large-amplitude response of antisymmetric angle-ply laminated rectangular plates subjected to broadband random excitation was studied by Mei and Wentz.[176] The formulation is based on the Kármán-type geometric nonlinearity, a single-mode Galerkin approach, the equivalent linearization method, and an iterative procedure. They considered angle-ply laminates of both simply supported and clamped-support conditions with either immovable or movable in-plane edges. Results obtained by them indicate that the presence of coupling between bending and extension in a laminate generally increases rms deflections and rms stresses and hence decreases the effective stiffnesses of a laminate.

Chonan[177] studied the dynamic response of an axially stressed, orthotropic cylindrical shell subjected to a step pressure pulse at its interior wall using a thick-shell theory in which the effects of the shear deformation and the rotatory and the longitudinal inertias were defined. Nath et al.[178] studied the nonlinear axisymmetric transient elastic stress and deflection responses of a cylindrically orthotropic thin circular plate with an elastically restrained edge, including both rotational and in-plane displacements. They employed in the analysis the dynamic analogue of the von Kármán governing differential equations in terms of the nodal displacement and stress function. The same authors in Ref. 179 performed a nonlinear axisymmetric transient analysis of orthotropic thin annular plates with a rigid central mass.

Matrix Methods and Eigenvalue Problems

Many researchers studied the wave propagation and transient response in composites by formulating an eigenvalue problem. There are several numerical techniques available to solve such problems. Yamada and Nasser[180] studied the dispersion of harmonic waves propagating in an arbitrary direction in a layered orthotropic elastic composite using the full three-dimensional field equations of elasticity and the corresponding twelfth-order characteristic determinant was examined.

Kausel[181] gave a unique semidiscrete method for studying the wave propagation problem in terms of an algebraic eigenvalue equation having a finite number of modes and involving narrowly banded matrices. Finding the natural modes of wave propagation in a layered anisotropic stratum normally requires the solution of intractable eigenvalue equations, which are characterized by a countable infinity of eigenvalues (wave numbers) and propagation modes. The method given by Kausel avoids the difficulties inherent in search procedures by resorting to a discretization of the displacement field in the direction of layering. Nelson et al.[182] presented a method for studying the free vibrations of a circular cylinder composed of an arbitrary number of cylindrically orthotropic layers. They formulated an eigenvalue problem by a discretized model from which the solutions are derived, given the natural frequencies and associated modal patterns.

Finite-Element Methods

Not much research has been carried out in the study of wave propagation in composites by a finite-element formulation. The advantage of well-established finite-element methods of structural analysis to study wave propagation in composites is its capability of handling more complex shapes of waveguides. Furthermore, when this method is employed to study the wave propagation in linear cases, the resultant equations of motion represent an eigenvalue problem, which can be solved by using many standard techniques.

Nassar and Nayfeh[183] applied the finite-element method to study the propagation of longitudinal elastic waves in laminated composites with bonds. They treated the geometric arrangement of the composite model as a special type of a trilaminated composite in which each of its major constituents is sandwiched between two bonding layers. The finite-element formulation includes 1) discretization of the actual composite waveguide into discrete finite elements, 2) mathematical formulation of the finite-element equation of motion, and 3) deriving the stiffness and inertia properties of the element. They proposed a linear two-dimensional plane strain element whose stiffness and mass matrices are functions of the number as well as the element geometrical and elastic properties.

Dong and Pauley[184] presented a finite-element method of analysis for the determination of frequencies and modal patterns of vibrations and waves in an infinite anisotropic plate. They expressed the waveform explicitly along the propagation direction and reserved the dependence through the thickness for a finite-element formulation. Teh and Huang[185] developed an analytical model based on elasticity equations to investigate wave propagation in generally orthotropic beams. They obtained approximate solutions to elasticity equations by the finite-element method. They applied the formulated finite-element model to determine the natural frequencies of 30 graphite/epoxy composite beams. They showed that neglecting the shearing-stress component contributed to the poor prediction of natural frequencies of a predominantly torsional mode for beams with a large thickness-to-width ratio. Huang and Dong[186] used the same finite-element modeling technique of Ref. 184 for investigating propagating waves and edge vibrations in isotropic composite cylinders. The finite-element modeling is in the radial direction, and thus the inhomogeneity in the form of uniform-thickness laminates is accommodated. They assumed harmonic functions for the spatial waveform in axial and circumferential directions. They determined the entire dispersive spectra from three different eigenvalue problems deducible from the same finite element.

A hybrid stress finite-element method was used by Feng et al.[187] in studying the dynamic response of simply supported laminated composite plates subjected to sinusoidal loading. The free vibrations were obtained using a space iteration method, and the dynamic response calculations were carried out using the Newmark direct-integration method.

Chen and Sun[188,189] studied the impact response of composite laminates with and without initial stresses using a nine-node isoparametric quadrilateral element based on the Mindin plate theory and von Kármán large-deflection strain–displacement relations. The authors incorporated an experimentally established contact law in their study. The contact law was capable of modeling permanent indentation. The governing equations were integrated using Newmark's constant acceleration algorithm in conjunction with successive iterations within each time step. The results for contact force histories, deflections, and the strains in the plate were presented.

Finally, numerical studies also have been conducted. Varadan et al.[190] developed a numerical technique to compute the effective elastic modulii attenuation and the phase velocity as a function of frequency and fiber concentration. They presented a multiple-scattering formalism using a T matrix to characterize the response of a single fiber to an incident wave so as to describe Primary (P-) and SV-wave propagation in a fiber-reinforced composite. A numerical procedure to study the rate of wave propagation in transversely isotropic materials was developed by Kline et al.[191] The waves were modeled as harmonic plane waves propagating in the plane of an infinite plate whose bounded surfaces were assumed to be stress free.

Concluding Remarks

The increasing use of laminated composite structural elements for a wide variety of applications in aerospace, mechanical, civil, and other branches of engineering has brought about an overwhelming interest for their analysis. This paper is a review of recent developments in the free-vibration analysis of symmetric and unsymmetric laminates; nonlinear vibrations of perfect and geometrically imperfect plates; and wave propagation (including linear and nonlinear transient response analysis) in laminated beams and plates. It is felt that this paper will be a valuable reference for the people engaged in the analysis and design of composite structures. This review brings together a large variety of research and developments in the analysis of laminated beams and plates and, we hope, will provide suggestions for future studies.

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